Worksheet 88 – Power Series

1) $\sum_{n=0}^{\infty}(x+5)^n$ $\quad c=-5$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(x+5)^{n+1}}{(x+5)^n}\right| = |x+5|$$

$|x+5|<1$

$-1<x+5<1$

$-6<x<-4$

$x=-6$: $\sum_{n=0}^{\infty}(-1)^n$ diverges by oscillation

$x=-4$: $\sum_{n=0}^{\infty}1^n$ diverges by n^{th} term

Radius $=1$

Interval: $-6<x<-4$

2) $\sum_{n=0}^{\infty}\frac{(x-2)^n}{10^n}$ $\quad c=2$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{10^{n+1}}\cdot\frac{10^n}{(x-2)^n}\right| = \left|\frac{x-2}{10}\right|$$

$-1<\frac{x-2}{10}<1$

$-10<x-2<10$

$-8<x<12$

$x=-8$: $\sum_{n=0}^{\infty}\frac{(-10)^n}{10^n}$ diverges by oscillation

$x=12$: $\sum_{n=0}^{\infty}\frac{10^n}{10^n}$ diverges by n^{th} term test

$R=10$

Interval: $-8<x<12$

3) $\sum_{n=0}^{\infty}\frac{nx^n}{n+2}$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(n+1)x^{n+1}}{n+3}\cdot\frac{n+2}{nx^n}\right| = |x|$$

$-1<|x|<1$

$-1<x<1$

$x=-1$: $\sum_{n=0}^{\infty}\frac{(-1)^n n}{n+2}$, $\lim_{n\to\infty}\frac{n}{n+2}=1$, diverges

$x=1$: $\sum_{n=0}^{\infty}\frac{n}{n+2}$, $\lim_{n\to\infty}\frac{n}{n+2}=1$, diverges

$R=1$

Interval: $-1<x<1$

4) $\sum_{n=0}^{\infty}\frac{3^n x^n}{n!}$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{3^{n+1}x^{n+1}}{(n+1)!}\cdot\frac{n!}{3^n x^n}\right| = \lim_{n\to\infty}\left|\frac{3x}{n+1}\right| = 0<1$$

$R=\infty$

Interval: $(-\infty,\infty)$

5) $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$ $\quad c=-3$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(n+1)(x+3)^{n+1}}{5^{n+1}}\cdot\frac{5^n}{n(x+3)^n}\right| = \left|\frac{x+3}{5}\right|$$

$$-1 < \frac{x+3}{5} < 1$$

$$-5 < x+3 < 5$$

$$-8 < x < 2$$

$x=-8$: $\sum_{n=0}^{\infty} \frac{n(-5)^n}{5^n}$ $\quad \sum_{n=0}^{\infty}(-1)^n n$ $\quad \lim_{n\to\infty} n = \infty$ diverges

$x=2$: $\sum_{n=0}^{\infty} \frac{n(5^n)}{5^n}$ $\quad \sum_{n=0}^{\infty} n$ $\quad \lim_{n\to\infty} n = \infty$ diverges

$R=5$

Interval: $-8 < x < 2$

6) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n\,2^n}$ $\quad c=-2$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x+2)^{n+1}}{(n+1)\cdot 2^{n+1}}\cdot\frac{n\,2^n}{(-1)^{n+1}(x+2)^n}\right|$$

$$= \left|\frac{x+2}{2}\right| < 1$$

$$-1 < \frac{x+2}{2} < 1$$

$$-2 < x+2 < 2$$

$$-4 < x < 0$$

$x=-4$: $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(-2)^n}{n\,2^n}$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{n}$ converges

$x=0$: $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n}$ converges

$R=2$

Interval: $-4 \le x \le 0$

7) $1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 + \cdots$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(x-3)^n}{2^n}$$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-3)^{n+1}}{2^{n+1}}\cdot\frac{2^n}{(-1)^n(x-3)^n}\right| = \left|\frac{x-3}{2}\right|$$

$$\left|\frac{x-3}{2}\right| < 1$$

$$-1 < \frac{x-3}{2} < 1$$

$$-2 < x-3 < 2$$

$$1 < x < 5$$

$x=1$: $\sum_{n=0}^{\infty}(-1)^{2n}$ diverges

$x=5$: $\sum_{n=0}^{\infty}(-1)^{n+1}$ diverges

$R=2$

Interval: $1 < x < 5$

8) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{(-1)^n (x^{2n+1})} \right| = \lim_{n\to\infty} \frac{x^2}{(2n+3)(2n+2)} = 0 < 1$$

$R = \infty$

Interval: $(-\infty, \infty)$

9) $\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x}{n+1} \right| = 0 < 1$$

$R = \infty$

Interval: $(-\infty, \infty)$

10) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{2n+2}}{(2n+2)!} \cdot \frac{(2n)!}{(-1)^n x^{2n}} \right| = \lim_{n\to\infty} \left| \frac{x^2}{(2n+2)(2n+1)} \right| = 0 < 1$$

$R = \infty$

Interval: $(-\infty, \infty)$